**Homework 8**

**P7.3.1** (a) Determine the equivalent capacitance between terminals ‘ab’ in Figure P7.3.1 if all the capacitances are1 F. (b) Determine the equivalent inductance between terminals ‘ab’, assuming that all capacitors in Figure P7.3.1 are replaced by inductors of 1 H.



**Solution:** (a) The middle combinations are equivalent to a 1 F capacitor each. Each branch becomes three 1 F capacitors in series, or 1/3 F. The two branches in parallel give 2/3 F.

(b) The middle combinations are equivalent to a 1 H inductor each. Each branch becomes three 1 H inductors in series, or 3 H. The two branches in parallel give 3/2 H.



**P7.3.5** (a) Determine the equivalent inductance between terminals ‘ab’ in Figure P7.3.5, assuming all inductances are 2 μH (b) Determine the equivalent capacitance between terminals ‘ab’, assuming that all inductors in Figure P7.3.5 are replaced by capacitors of the same value in μF.

**Solution:** (a) Two 2 μH inductors in parallel are equivalent to 1 μH. This in series with 2 μH is 3 μH. In parallel with 1 μH, the equivalent inductance is ¾ μH.



(b) Two 2 μF capacitors in parallel are equivalent to 4 μF. This in series with 2 μF is 4/3 μF. In parallel with 4 μF, the equivalent capacitance is 16/3 μF.

**P7.4.5** (a) Derive the dual of the circuit of Figure P7.4.5; (b) represent the two circuits in the dc steady state; (c) compare voltage division in given circuit with current division in dual circuit; (d) compare the power delivered or absorbed by each circuit element in the two circuits.



**Solution:** (a) The dual of 15 V in series with three branches is 15 A in parallel with duals of the series branches. The dual of 10 Ω is 10 S, the dual of 10 μH is 10 μF, and the dual of 2 μF in parallel with 20 Ω is 2 μH in series with 20 S.



(b) The two circuits are shown in the dc state, whereby, inductors act as short circuits and capacitors as open circuits.



(c) From voltage division in the given circuit, the voltage across the 10 Ω resistance is 5 V, and that across the 20 Ω resistance is 10 V. From current division in the dual circuit, the current through the 10 S conductance is 5 A, and that through the 20 S conductance is 10 A.

(d) The current through the 15 V source is 0.5 A, and the voltage across the 15 A source is 0.5 V. The power delivered in each case is 7.5 W. The 10 Ω resistor dissipates 25/10 = 2.5 W, and the 10 S resistor dissipates 25/10 = 2.5 W. The 20 Ω resistor dissipates 100/20 = 5 W, and the 20 S resistor dissipates 100/20 = 5 W.

**P8.1.6** Given **A** = 3 + *j*5, **B** = 10 – *j*8, and **C** = *j*12. Determine the phasors resulting from the following operations: (a) **A**\***B**\***C**; (b) (**A**\***B**)/**C**; (c) (**A**/**B**)\***C**; and (d) **A**/**B**/**C**. Express the result in rectangular and polar forms.

**Solution:** Using Matlab:

A=3+*j*5, B=10–*j*8, C=*j*12.

1. a=A\*B\*C gives -312 + *j*840.

[tha,ra]=cart2pol(real(a),imag(a)) gives 1.9264, 896.1 ≡ 896.1∠110.4°.

1. b=(A\*B)/C gives 2.1667 – *j*5.8333.

[thb,rb]=cart2pol(real(b),imag(b)) gives –1.2152, 6.223 ≡ 6.223∠-69.62°.

1. c=(A/B)\*C gives -5.4146 – *j*0.7317.

[thc,rc]=cart2pol(real(c),imag(c)) gives –3.0073, 5.464 ≡ 6.223∠-172.3°.

1. d=(A/B)/C gives 0.0376 + *j*0.0051.

[thd,rd]=cart2pol(real(d),imag(d)) gives 0.1343, 0.0379 ≡ 0.0379∠7.69°.

**P8.1.9** Using phasors, determine the steady-state *y* that satisfies the differential equation:



Express *y* as a cosine time function (Hint: express the RHS as a phasor).

**Solution:** The problem will be solved using phasor notation. The RHS is , where , or in phasor notation . Differentiation is equivalent in the steady state to multiplying by *jω*, or *j*4, and integration is equivalent to dividing by *j*4. The LHS becomes (*j*24 + 3 – *j*/2)**Y** = , where the angle is in radians. Hence, **Y** =  , or 0.472∠–146.2°. As a cosine function of time this is 0.472cos(4*t* – 146.2°). Using phasors is equivalent to ignoring the constant in the integration on the LHS.